

THURSDAY, APRIL 23, 1903.

SCHOOL GEOMETRY REFORM.

Practical Exercises in Geometry. By W. D. Eggar, M.A. Pp. xii+287. (London: Macmillan and Co., Ltd., 1903.) Price 2s. 6d.

Geometry. An Elementary Treatise on the Theory and Practice of Euclid. By S. O. Andrew, M.A. Pp. xi+182. (London: John Murray, 1903.) Price 2s.

Theoretical Geometry for Beginners. By C. H. Allcock. Pp. ix+135. (London: Macmillan and Co., Ltd., 1903.) Price 1s. 6d.

Elementary Geometry. By W. M. Baker, M.A., and A. A. Bourne, M.A. Books i. and ii., pp. xxix+126; price 1s. 6d. Books i.-iii., pp. xxix+213; price 2s. 6d. Books i.-iv., pp. xxix+272; price 3s. Books i.-vii., pp. xxix+474; price 4s. 6d. (London: George Bell and Sons, 1903.)

The Elements of Geometry. By R. Lachlan, Sc.D., and W. C. Fletcher, M.A. Pp. xii+207. (London: Edward Arnold, n.d.) Price 2s. 6d.

Plane Geometry. Adapted to Heuristic Methods of Teaching. By T. Petch, B.A. Pp. vii+112. (London: Edward Arnold, n.d.) Price 1s. 6d.

Euclid: Books v., vi., xi. By Rupert Deakin, M.A. Pp. 144. (London: W. B. Clive, 1903.) Price 1s. 6d.

A Short Introduction to Graphical Algebra. By H. S. Hall, M.A. Pp. 49. (London: Macmillan and Co., Ltd., 1903.) Price 1s.

THE movement having for its object the improvement of the teaching of elementary geometry is making rapid progress; witness the enthusiastic support of the teachers, the adhesion of important examining bodies, and the number of new text-books now appearing in rapid succession.

In the "Practical Exercises in Geometry," by Mr. W. D. Eggar, we have a contribution of remarkable freshness. In this valuable text-book the method pursued is on lines indicated long ago by W. G. Spencer, the father of Mr. Herbert Spencer, in his "Inventional Geometry,"¹ a little work that should be known to all teachers. The principal advance on Spencer's geometry is in the amount of *quantitative measurement* introduced, and in the use of squared paper methods. The author describes his book as "an attempt to adapt the experimental method to the teaching of geometry in schools." He says:—

"The main object of this method, sometimes called 'heuristic,' is to make the student think for himself, to give him something to do with his hands for which the brain must be called in as a fellow-worker. The plan has been tried with success in the laboratory, and it seems to be equally well suited to the mathematical class-room."

And readers of the book will agree that the author has very good grounds for this opinion.

The first five chapters are devoted entirely to the measurements of lines, arcs and angles. The author wisely uses only decimal scales. These are the inch and the centimetre scales; in regard to the latter it is no small advantage for a youth to be trained so as to be able to think in metric units. The degree of

accuracy aimed at will appear from the requirement that students are asked to measure lengths correctly to within the one-hundredth part of an inch. This, however, will prove to be rather trying for lines in some of the figures, in the absence of short cross lines defining their ends. Several methods are suggested of how to measure the circumference of a circle, but the use of tracing paper and a pricker, perhaps the best of these, is overlooked.

The student is next introduced to the use of set-squares, and the notions of parallel and perpendicular lines naturally follow. Explanations are then given of how areas and volumes are measured, the subject being illustrated by the use of squared paper, unit cubes, graduated flasks, weighing, &c. The quantitative work is here largely arithmetical. This free admixture of arithmetic and drawing is, in fact, a feature throughout the book, and one marvels at the long unnatural divorce which has existed between the two in the past.

Chapters xi. and xii. are devoted to some fundamental constructions, such as the bisection of lines and angles, and the division of lines. The student by this time is quite familiar with the notion of a locus.

So far the work has been more or less of preparation. The student is now led to study more particularly the properties of triangles, quadrilaterals, circles, proportionals and similar figures. But there is no change in the method of treatment. By judicious directions, by questions and suggestions, the reader all the while seems to be discovering new truths for himself by drawing and measurement, and his interest is secured and maintained. Then follows the reason, given quite informally, perhaps by a mere hint, but none the less perfectly logical, and absolutely convincing and satisfactory, and the student feels that he has, or could have, discovered this also.

The concluding chapters relate to mensuration rules, the graphical solution of quadratic equations, the construction of scales, and graphs.

Material is provided at the ends of some of the chapters for the student to exercise himself in riders, constructions, and numerical examples. The answers to the latter are collected at the end of the volume.

The course above outlined is developed on satisfactory lines, and may be regarded as a first important instalment to the new literature of the subject. Taught in this manner, geometry would seem likely to become the most popular, as well as the most illuminating branch of elementary mathematics. It ought to replace not only Euclid, but the wretched system of practical plane geometry now in vogue in our elementary day schools. The course includes everything contained in the first six books of Euclid that a boy need know; and he knows it so thoroughly that any subsequent study of Euclid or its equivalent will add little to his knowledge of geometry, whatever may be its other merits or demerits.

We notice that the use of the T-square is not introduced at all. This seems a pity, in view of its great utility and of future developments.

While in general agreement with the author, we should like to see his course of study extended. What-

¹ Published by Williams and Norgate.

ever may be added, however, should be carefully selected, having regard to modern conditions. Very little additional matter will be taken from Euclid. We think the book would have been improved by a chapter on the solution of right-angled triangles, using the trigonometrical tables given at the end of the volume, results obtained graphically being verified by calculation. In subsequent work graphical and numerical computation would go on side by side. There are calculations relating to right-angled triangles quite as important as that of *Euc. i. 47*, and the drawing class seems to be the proper place in which to teach them to beginners. What better examples than the trigonometrical functions are to be found of ratio and proportion? Consider what a satisfaction it must be to a boy to find himself in possession of and familiar with this powerful modern weapon. And, moreover, the knowledge gained is of the utmost importance. In connection with this part of the subject, the radian measure of an angle should not be neglected; it is very desirable that a student should be trained so as to be able to think in radians as well as in right angles and degrees.

Next, a course seems very incomplete without some notion of projection, and how lengths and angles in three dimensional space are measured and represented. Following the author's plan, the principles of *Euclid xi.* would be inculcated along with exercises in descriptive geometry, involving quantitative measurement. This can be rendered quite interesting.

And lastly, one of the most fruitful additions that could possibly be made would be to introduce the idea of a vector, giving the triangle or parallelogram law, with some of its consequences. Geometry is essentially a vector subject, and an early knowledge of vectors would have far-reaching effects.

In the "Geometry" by Mr. S. O. Andrew we have another text-book in which exercises in drawing and deductive reasoning are carried on together, so that the student acquires some practical acquaintance with the subject-matter. But the work is not based sufficiently on accurate quantitative measurement, and the author seems satisfied with drawing of an inferior quality. We find no description of what sort of scales are suitable for measuring lengths. There is no information as to the manner of using and testing straight edges and squares. In the absence of any guidance to the contrary, the student is sure to use soft blunt pencils. There are no numerical answers given to any of the exercises.

But a teacher using the book could, to some extent, supply these omissions, and would find the volume very serviceable; it is the result of practical teaching experience. It covers substantially the same ground as the book previously considered, with a chapter on solid geometry and orthographic projection. Loci and graphs are introduced, and trigonometrical tables are given and explained, but are made very little use of in the text.

The text-books of Messrs. Allcock, Baker and Bourne, Lachlan and Fletcher, and Petch are alike in having for their main object the development of a system of formal geometry on Euclidean lines. The

changes they introduce with the object of improving geometrical teaching are such alterations as the revision of the definitions and axioms, the rearrangement and regrouping of the propositions, the employment of arithmetic, algebra, loci, &c. Euclid's *form* of reasoning has in all cases been retained. Experimental geometry is not made prominent; it is brought in rather in connection with the examples which follow the propositions.

As it appears to us, these books are not sufficiently free of the Euclidean tradition to make them suitable for boys at school. They are more fitted for subsequent study. The presentation of the substance of *Euclid i.* by Allcock is excellent, and may well replace Euclid when the time comes for taking up the philosophy of the subject. In the volumes by Messrs. Baker and Bourne there is an introductory chapter on experimental geometry, extending over twenty pages, comprised of nearly two hundred exercises, ranging over the whole subject up to the end of *Euclid vi.*, and intended to make the student practically acquainted with the ground to be subsequently covered. This chapter is a valuable and extremely suggestive one, so far as it goes; if the material had been set out in greater detail, and worked in along with the deductive geometry and accorded equal importance with the latter, a geometry quite suitable for youths would have been the result. As text-books of formal geometry these manuals by Messrs. Baker and Bourne can be strongly recommended. They cover the ground usually studied, including *Euclid xi.*, and there are chapters on graphs and mensuration formulæ. They are beautifully printed and arranged, and contain many practical exercises.

Mr. Deakin's *Euclid* is written on strictly orthodox lines; it contains some useful notes and exercises by the author. The only evidence of any influence of the reform movement is at the end of *Euclid vi.*, where an abstract is given of the recommendations of the committee of the Mathematical Association of 1902.

The little book on "Graphical Algebra" by Mr. Hall is intended to accompany the well-known "Elementary Algebra" of Messrs. Hall and Knight. It is concerned with graphs and squared paper work, and illustrates some part of the service which geometry is rendering to algebra. Some of the examples are evidently taken from previous publications, though the author forgets to acknowledge their source.

J. HARRISON.

SYSTEMATIC PETROGRAPHY.

Quantitative Classification of Igneous Rocks Based on Chemical and Mineral Characters, with a Systematic Nomenclature. By Whitman Cross, Joseph P. Iddings, Louis V. Pirsson, and Henry S. Washington, with an Introductory Review of the Development of Systematic Petrography in the Nineteenth Century, by Whitman Cross. Pp. x + 286. (Chicago: the University of Chicago Press; London: Wm. Wesley and Co., 1903.) Price 8s. net.

BY the very first page this book is defined as dealing with "the science of petrography." Petrology is "the broad science or treatise of rocks"; petro-